

2

# NAVAL POSTGRADUATE SCHOOL

Monterey, California

AD-A225 491



DTIC FILE COPY

## THESIS

DTIC  
ELECTRONIC  
AUG 20 1990  
D  
E

AN APPROACH TO AN ASSIGNMENT PROBLEM  
WITH HIERARCHICAL OBJECTIVES

by

Michael S. Paul

March, 1990

Thesis Advisor:

S. Lawphongpanich

Approved for public release; distribution is unlimited

Unclassified

## SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE				Form Approved OMB No. 0704-0188	
1a. REPORT SECURITY CLASSIFICATION UNCLASSIFIED			1b. RESTRICTIVE MARKINGS		
2a. SECURITY CLASSIFICATION AUTHORITY			3. DISTRIBUTION/AVAILABILITY OF REPORT Approved for public release; distribution is unlimited		
2b. DECLASSIFICATION/DOWNGRADING SCHEDULE					
4. PERFORMING ORGANIZATION REPORT NUMBER(S)			5. MONITORING ORGANIZATION REPORT NUMBER(S)		
6a. NAME OF PERFORMING ORGANIZATION Naval Postgraduate School		6b. OFFICE SYMBOL 55		7a. NAME OF MONITORING ORGANIZATION Naval Postgraduate School	
6c. ADDRESS (City, State, and ZIP Code) Monterey, CA 93943-5000			7b. ADDRESS (City, State, and ZIP Code) Monterey, California 93943-5000		
8a. NAME OF FUNDING/SPONSORING ORGANIZATION		8b. OFFICE SYMBOL		9. PROCUREMENT INSTRUMENT IDENTIFICATION NUMBER	
8c. ADDRESS (City, State, and ZIP Code)			10. SOURCE OF FUNDING NUMBERS		
			PROGRAM ELEMENT NO.	PROJECT NO.	TASK NO.
			WORK UNIT ACCESSION NO.		
11. TITLE (Including Security Classification)  An Approach to an Assignment Problem with Hierarchical Objectives					
12. PERSONAL AUTHOR(S) PAUL, Michael S.					
13. TYPE OF REPORT Master's Thesis		13b. TIME COVERED FROM TO		14. DATE OF REPORT (Year, Month, Day) 1990, March	
15. Page Count 57					
16. SUPPLEMENTAL NOTATION The views expressed in this thesis are those of the author and do not reflect the official policy or position of the Department of Defense or the U.S. Government.					
17. COSATI CODES			18. SUBJECT TERMS (Continue on reverse if necessary and identify by block number)		
FIELD	GROUP	SUB-GROUP	Multiobjective programming, assignment problem, noninferior set estimation, Navy Officer Assignment, Manpower planning		
19. ABSTRACT (Continue on reverse if necessary and identify by block number) At the Naval Military Personnel Command (NMPC), multiple objectives must be considered in assigning personnel to billets. For the assignment of Naval officers, these objectives in decreasing order of importance are to satisfy the needs of the Navy, to enhance the careers of officers, to fulfill the desires of officers, and to minimize cost.  To assist in this complicated task, a procedure which considers these four objectives in their order of importance is proposed. Each time, a standard assignment problem is solved by optimizing one objective with the additional constraint that values of the other more important objectives remain above specified levels. A modification of a multiobjective programming technique, the Noninferior Set Estimation method, is used to guarantee integer solutions to an assignment problem with these additional constraints.  An application of the procedure to an actual Navy officer assignment problem indicates its potential as a decision aid to NMPC officers and other decision makers.					
20. DISTRIBUTION/AVAILABILITY OF ABSTRACT <input checked="" type="checkbox"/> UNCLASSIFIED/UNLIMITED <input type="checkbox"/> SAME AS RPT. <input type="checkbox"/> DTIC			1a. ABSTRACT SECURITY CLASSIFICATION Unclassified		
22a. NAME OF RESPONSIBLE INDIVIDUAL Siriphong Lawphongpanich			22b. TELEPHONE (Include Area Code) (408)646-2106		22c. OFFICE SYMBOL 55Lp

Approved for public release; distribution is unlimited.

An Approach  
to an  
Assignment Problem with Hierarchical Objectives

by

Michael S. Paul  
Lieutenant, United States Navy  
B.S.E., University of Michigan

Submitted in partial fulfillment  
of the requirements for the degree of

MASTER OF SCIENCE IN OPERATIONS RESEARCH

from the

NAVAL POSTGRADUATE SCHOOL  
March 1990

Author:



Michael S. Paul

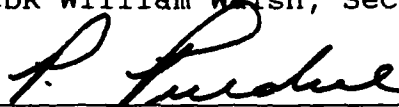
Approved by:



Siriphong Lawphongpanich, Thesis Advisor



LCDR William Walsh, Second Reader



Peter Purdue, Chairman  
Department of Operations Research

## ABSTRACT

At the Naval Military Personnel Command (NMPC), multiple objectives must be considered in assigning personnel to billets. For the assignment of Naval officers, these objectives in decreasing order of importance are to satisfy the needs of the Navy, to enhance the careers of officers, to fulfill the desires of officers, and to minimize cost.

To assist in this complicated task, a procedure which considers these four objectives in their order of importance is proposed. Each time, a standard assignment problem is solved by optimizing one objective with the additional constraint that values of the other more important objectives remain above specified levels. A modification of a multiobjective programming technique, the Noninferior Set Estimation method, is used to guarantee integer solutions to an assignment problem with these additional constraints.

An application of the procedure to an actual Navy officer assignment problem indicates its potential as a decision aid to NMPC officers and other decision makers.

Accession For	
NTIS GPO	<input checked="checked" type="checkbox"/>
DTIC TAB	<input type="checkbox"/>
Unannounced	<input type="checkbox"/>
Justification	
By	
Distribution/	
Availability Codes	
Dist	Avail and/or Special
A-1	

## TABLE OF CONTENTS

I.	INTRODUCTION . . . . .	1
	A. THE TRIAD OF DETAILING . . . . .	2
	B. THE FOURTH OBJECTIVE . . . . .	4
	C. THE ASSIGNMENT PROCESS . . . . .	5
	D. AN EXISTING MODEL FOR ENLISTED PERSONNEL . . .	8
	E. AN ALTERNATE APPROACH . . . . .	10
II.	A HIERARCHICAL APPROACH . . . . .	12
	A. THE HIERARCHICAL SOLUTION PROCESS . . . . .	13
	B. THE ASSIGNMENT PROBLEM WITH SIDE CONSTRAINTS .	15
III.	A MULTIOBJECTIVE SCHEME FOR THE NOA PROBLEM . . .	17
	A. EFFICIENT POINTS AND FRONTIERS . . . . .	18
	B. AN ALGORITHM FOR SOLVING AN ASSIGNMENT PROBLEM WITH SIDE CONSTRAINTS . . . . .	24
	C. A BIOBJECTIVE EXAMPLE PROBLEM . . . . .	30
IV.	EXAMPLE PROBLEM . . . . .	35
	A. DATABASE DESCRIPTION . . . . .	36
	B. IMPLEMENTATION OF THE MODIFIED NISE ALGORITHM .	40
V.	CONCLUSION . . . . .	45

LIST OF REFERENCES . . . . .	48
------------------------------	----

INITIAL DISTRIBUTION LIST . . . . .	50
-------------------------------------	----

## I. INTRODUCTION

It has always been the objective of the United States Navy to obtain the maximum utilization of its officers and enlisted personnel. This maximum utilization is realized by assigning the "best" person to the "best" job or billet. The decision of what is the "best" assignment depends on many factors that make this an interesting, but controversial assignment problem.

Many considerations must be made to achieve a good match between an officer or an enlisted person and a billet. For an officer, the individual of interest for this thesis, these considerations include warfare designation, rank, additional qualification designations (AQD), subspecialties acquired, fitness reports, planned rotation date (PRD), billet priority, career path, desires, and cost. The Navy's policy of rotating personnel from sea to shore billets and shore to sea billets adds an additional constraint to an already complicated problem.

The personnel assignment policy of the Naval Military Personnel Command (NMPC) that takes into account these many considerations involves three primary objectives. These objectives are called the "Triad of Detailing" (NMPC-4, 1982, pp. 2:1-2).

## A. THE TRIAD OF DETAILING

The "Triad of Detailing" consists of three objectives: (1) the needs of the Navy, (2) the career and qualification needs of the officer, and (3) the desires of the officer. Ideally, the weight of each of the above objectives would be equal for an assignment, but often they are not. The needs of the Navy has priority in such cases and overrides all other factors. An example would be the assigning of an officer with an aviation warfare designation to a training command as an instructor. This individual may desire assignment to the Naval Postgraduate School, a career enhancing billet, but the scarcity of instructors requires the officer to be assigned to the training command. Identifying the needs of the Navy as the primary factor ensures that the billets allowed to each activity are filled by the best officer.

Certain skills and qualifications are scarce, as indicated in the above example. An individual having such resources should be utilized accordingly. The assignment of a specially skilled officer to a billet not requiring his or her skills is a waste of valuable resources and should be avoided.

The best officer for a job may not be available because of the sea-shore rotation policy or an officer's PRD. Leaving a billet unoccupied in anticipation of a certain officer does occur, but this is often undesirable. Instead



of searching the officer database for officers with certain skills to fill a billet, the officers available near the time of vacancy of a billet are considered. The best available officer is selected, keeping vacant billets to a minimum. Meeting the needs of the Navy can thus be viewed as filling high priority billets, usually requiring special skills, with the best available officer in a timely fashion.

The second most important objective of the "Triad of Detailing", the career and qualification needs for officers, is important for two reasons: (1) a carefully planned career path keeps officers on track for promotions and command opportunities, and (2) long range planning for future needs of the Navy must be considered to prevent scarcity of certain skills and qualifications.

Meeting the desires of an individual, the third objective of the "Triad of Detailing," is important for both morale and retention. The officer duty preference card is used by officers to express their desires of activity, homeport, and billet type to their NMPC representative, the Assignment Officer (AO). Every effort is made by the AO to assign the officer to a billet that meets some or all of the desires of an individual, as long as these desires do not conflict with the first two objectives of the "Triad of Detailing".

## B. THE FOURTH OBJECTIVE

Absent from the "Triad of Detailing" is the cost of an assignment. From NMPC's point of view, the major component of this cost is the Permanent Change of Station (PCS) cost which consists of the expense associated with relocating officers and their families. In the past, because of the already complicated task faced by the AO in making the assignment and perhaps a relatively liberal budget, the cost of an assignment was not considered. However, in the current cost conscious environment, reductions in the defense budget make the cost of an assignment a critical issue. In fact, the United States Navy was forced to hold its personnel at their current assignments in January of 1988 in order to remain within the Permanent Change of Station (PCS) budget.

In light of the current availability and power of computers, the assignment process can be partially automated, thereby reducing the complexity of the task. Since adding a fourth objective to this automated process does not appreciably increase the complexity as far as the two users, the Placement and Assignment officers, are concerned, it is not only appropriate but also beneficial to include cost along with the triad of detailing. It is hoped that by considering cost in the assignment process, the U.S. Navy will never have to resort to such drastic measures as holding personnel in their current billets again.

### C. THE ASSIGNMENT PROCESS

Three key individuals involved in the assignment of an officer to a billet are: the Placement Officer (PO), the Assignment Officer (AO), and the Naval officer awaiting assignment. Each of these officers essentially represents one of the objectives in the "Triad of Detailing".

The PO has cognizance over a number of activities, such as ships or squadrons, and the officers assigned to them. These activities are made up of allocated billets. The PO is directly responsible to Commanding Officers of these activities for the manning of their billets. This officer will have usually been assigned to one of these activities earlier in his career and therefore will have an understanding of their assignment needs and problems. The PO attempts to find the best available officer for his or her activities to satisfy the needs of the Navy.

The AO is a representative of a group of constituent officers. The main responsibility of an AO is to ensure that the constituent officers gain the required knowledge and experience to take on command responsibilities and to meet long range Navy needs. Typically, an AO should have a similar primary designator and/or additional qualification designator (AQD) as the constituent officers in order to effectively guide officers along their career paths. The AO, then, represents the career and qualification needs for officers, the second objective of the "Triad of Detailing".

The last objective in the triad concerns an officer's preference for the available billets. Officers indicate their desire for billets on the duty preference cards (see Figure 1). The completed cards are reviewed by the AO for assignment considerations. It is possible that the AO may request that the officer change his or her preference if it would jeopardize the officer's career path.

The assignment process begins when the PO posts billets to be filled. These billets usually become "vacant" as the incumbent officers move to new billets at the end of their tour. A group of these billets may have a high priority and cannot be left unfilled during the current assignment process. In this situation, the PO may elect to post only the high priority billets first. This action ensures these billets will be filled before those billets with lower priority are considered.

Given the posted billets, the AO then nominates an officer from his constituency to each of the posted billets. The PO examines these nominations and may accept or reject them. For those billets where the nominations are rejected, further negotiations may result in an acceptance or a new officer being nominated to the billet. It is during the negotiations between the PO and AO that "values", "weights", or utilities are implicitly assigned to the three objectives in the triad. These values, weights, or utilities cannot be numerically expressed nor quantified. However, they should

<b>OFFICER PREFERENCE AND PERSONAL INFORMATION CARD</b>									
<b>1 GENERAL INFORMATION</b>									
SOCIAL SECURITY NUMBER			NAME (LAST, FIRST, MIDDLE IN BLOCKS) PLACE FIRST 5 LETTERS OF LAST NAME IN BLOCKS				SUBMIT MONTH		
UIC			AUTOVON PHONE		AREA CODE		COMMERCIAL PHONE		GRADE
AREA CODE			HOME PHONE		HOME OF RECORD		STATE		PRIMARY DUTY
HOME ADDRESS (RR, STREET ADDRESS, APT. #, ETC.)									
CITY			STATE		ZIP		USNR OFFICER ONLY RAE MO YR REASSIGNMENT DESIRED AT RE		Y OR N <input type="checkbox"/>
<b>2 NEXT DUTY PREFERENCES AND PRIORITIES</b>									
HOMEPORT LOCATION			LOCATION CODE		SHIP SQUAD A, T, Y		TYPE OF BULLET		
1			1			1			
2			2			2			
3			3			3			
4			4			4			
5			5			5			
6			6			6			
<b>3 POST GRADUATE PREFERENCE</b>									
ENTER 3 DIGIT CURRICULUM NUMBER FROM MILPERSMAN TITLE 10			IF POSTGRADUATE NOT DESIRED, PLACE AN 'X' IN THE BOX						
1ST CHOICE			2ND CHOICE			3RD CHOICE			
<b>4 DEPENDENT INFORMATION</b>									
M MARRIED			MILITARY SPOUSE SSN (IF NAVY)						
W WORKING SPOUSE			BRANCH OF SERVICE (IF OTHER)						
N NOT MARRIED									
S SINGLE PARENT									
MILITARY APT. LOCATION CODE			YR		NUMBER AND AGES OF MINOR DEPENDENTS RESIDING WITH SERVICE MEMBER				
					M F M F M F				
					0 4 5 11 12+				
					NUMBER OF SECONDARY DEPENDENTS				
<b>5 SIGNIFICANT MODELS FLOWN</b>									
MODEL		HOURS	YR LAST FLOWN	CV	PPC MC TIME	MODEL		HOURS	YR LAST FLOWN
1						3			
2						4			
<b>6 REMARKS</b>									
EVERY OFFICER SHALL SUBMIT ONE ORIGINAL AND ONE COPY TWELVE MONTHS PRIOR TO PRE AND WHEN SIGNIFICANT CHANGES OCCUR. SEE MILPERSMAN ART 182140. TYPE OR PRINT WITH BLUE OR BLACK INK. WHEN COMPLETED, SEND TO INDIVIDUAL ASSIGNMENT OFFICER, NAVAL MILITARY PERSONNEL COMMAND, WASHINGTON, DC 20370.									

NAVPERS 1301/1 (REV 10-83)

SN 0106 LF 013 0106

**Figure 1: Officer Preference Card.**

i.e., Navy needs should have more weight than career and qualification needs which should have more weight than desires.

#### **D. AN EXISTING MODEL FOR ENLISTED PERSONNEL**

Currently, there is no automated system or computer program to assign Navy officers to billets. However, the Navy Personnel Research and Development Center (NPRDC) has developed a computer program called the Enlisted Personnel Allocation and Nomination System (EPANS) to assign enlisted personnel to billets.

Similar to the "Triad of Detailing", there are multiple objectives which must be considered in assigning the enlisted personnel to billets. Presently EPANS considers approximately 14 objectives. The large number of objectives is due to the developer's conception of the objectives of NMPC's assignment policy. For example, in EPANS, the Navy needs objective is broken into three separate objectives: (i) billet eligibility, (ii) timeliness of assignment, and (iii) billet priority.

To deal with the large number of objectives, EPANS combines all objectives into a single (combined) objective which is the weighted sum of all 14 objectives (see Liang and Thompson, 1987). It is clear that the weights assigned to the objectives affect the assignment. To select these

weights, NPRDC conducted statistical studies and interviews with NMPC personnel to confirm the statistical results. From personal experience at NMPC and conversations with Dr. Liang, there is no consensus as to what the weights should be. EPANS currently allows users to specify or use default values for the weights applied to the objectives. However, NMPC plans to "standardize" the weights in the future.

When faced with multiple objectives, the approach of reducing them into one combined objective through the use of weights was once popular because of its simplicity and ease in implementation. However, many researchers have discovered that the results from the weighting scheme do not necessarily resemble the user's expectation. As an example, Steuer and Schuler (1978) experimented with this weighting approach in the multiobjective forestry problem and reported the following:

The results were exasperating. Sample weighting vectors, tempered by the above five considerations, that appeared to correspond well with the forest-planners' intentions were found to produce poor points; while other sample vectors that did not even appear to resemble the forest planners' purposes (e.g., with a penalty weight of zero being placed upon deviations from the most important goal) led to much better points.

The primary goal of this thesis is to present an alternate method to the weighting scheme.

## E. AN ALTERNATE APPROACH

In trying to eliminate the possibility of producing results contrary to the user's expectations, the approach taken in this thesis does not use weights. Instead, the objectives are considered one at a time in the order of their importance (or hierarchy) as indicated by the "Triad of Detailing". This approach is viable since there are only four objectives, the triad plus the cost, in the officer assignment model.

This hierarchical approach first computes assignments which maximize the most importance objective, Navy needs. Given the maximal level of Navy needs, the method then finds assignments which maximize the second most important objective, career and qualification needs of the officers, while maintaining an acceptable percentage, e.g. 95%, of the maximal level of Navy needs. Given the maximal levels of the first and second important objectives computed thus far, the method next calculates a third set of assignments which maximize the third objective, desires, while maintaining acceptable percentages of maximal levels of Navy needs and career needs. The process continues in the same manner for the fourth objective, cost.

In Chapter II, the Navy assignment model is formulated, implying a multiobjective methodology based on efficiency as its solution. Chapter III reviews the multiobjective concept of efficiency and presents an approach, a



modification of the Noninferior Set Estimation (NISE) Procedure, that uses this concept to solve an assignment problem with hierarchical objectives. Chapter IV illustrates the approach found in Chapter III by solving an example Navy officer assignment problem. Chapter V, the final chapter, presents a summary of the approach and conclusions.

## II. A HIERARCHICAL APPROACH

The Navy officer assignment problem can be modeled as a mathematical problem with multiple objectives as follows:

### The Navy Officer Assignment (NOA) Problem

#### Indices

i officers  
j billets

#### Data

$C_{ij}$  level of Navy needs being satisfied by assigning officer i to billet j

$D_{ij}$  level of career needs being satisfied by assigning officer i to billet j

$E_{ij}$  degree of desire expressed by officer i for billet j

$F_{ij}$  cost of assigning officer i to billet j

#### Decision Variable

$x_{ij} = 1$ , if officer i is assigned to billet j  
= 0, otherwise

#### Objectives (in hierarchical order)

$$\text{MAXIMIZE } \left\{ \begin{array}{ll} f_1(\mathbf{x}) = \sum_{i,j} C_{ij}x_{ij} & \text{Navy Needs} \\ f_2(\mathbf{x}) = \sum_{i,j} D_{ij}x_{ij} & \text{Career} \\ f_3(\mathbf{x}) = \sum_{i,j} E_{ij}x_{ij} & \text{Desires} \\ f_4(\mathbf{x}) = \sum_{i,j} -F_{ij}x_{ij} & \text{(Negative) Cost} \end{array} \right\}$$

where  $x$  is a vector of  $x_{ij}$  values.

### Constraints

$$\sum_j x_{ij} = 1, \quad \forall i$$
$$\sum_i x_{ij} = 1, \quad \forall j$$

The first constraint ensures that each officer is only assigned to one billet and the second constraint ensures that each billet is only assigned to one officer. Implicitly assumed is the fact that the number of available officers equals the number of available billets. However, the constraints can be easily modified to accommodate other variations of the above model.

### **A. THE HIERARCHICAL SOLUTION PROCESS**

Although the NOA problem can be addressed in the framework of multiobjective programming, treating the objectives as being on four different hierarchical levels more accurately models the process as it exists today. The most important objective in the triad, the Navy needs objective function,  $f_1(x)$ , is first maximized without regard to the other three objectives. This maximization corresponds then to the following problem:

**PROBLEM 1:**  $f_1^* = \max f_1(x)$   
s.t.

$$\sum_j x_{ij} = 1, \quad \forall i$$
$$\sum_i x_{ij} = 1, \quad \forall j$$
$$x_{ij} \in \{0,1\}$$

where  $f_1^*$  represents the maximum level of Navy needs that can be satisfied with the available officers. The next step in the process is to account for the other objectives in the triad according to their hierarchical order.

Second in the hierarchy is the level of career and qualification needs. Typically, there is a trade-off between the level of Navy and career needs. It would be naive to expect an increase in the level of Navy needs to trigger an increase in the level of career needs. So, in order to raise the level of career needs, the level of Navy needs must be degraded. It is then assumed that the decision makers, i.e., the assignment and placement officers, have already agreed on an acceptable level of degradation,  $(1-p_1)$ , for the level of needs, where  $p_1 \in (0,1)$ . For example, if the level of degradation was 5%, then  $p_1$  would be .95. Given this decision parameter, the problem is then to maximize the level of career needs while holding the level of Navy needs to an acceptable level. This corresponds to the following problem:

$$\begin{aligned}
 \text{PROBLEM 2: } f_2^* &= \max f_2(\mathbf{x}) \\
 \text{s.t. } & \sum_j x_{ij} = 1, \quad \forall i \\
 & \sum_i x_{ij} = 1, \quad \forall j \\
 & f_1(\mathbf{x}) \geq p_1 f_1^* \\
 & x_{ij} \in \{0,1\}
 \end{aligned}$$

Similarly, to account for the remaining two objectives, desires and cost, the level of degradation for objectives higher in the hierarchy must be specified and the following mathematical problems must be solved in sequence:

$$\begin{aligned} \text{PROBLEM 3: } f_3^* &= \max f_3(\mathbf{x}) \\ \text{s.t. } & \sum_j x_{ij} = 1, \quad \forall i \\ & \sum_i x_{ij} = 1, \quad \forall j \\ & f_k(\mathbf{x}) \geq p_k f_k^*, \quad k=1,2 \\ & x_{ij} \in \{0,1\} \end{aligned}$$

$$\begin{aligned} \text{PROBLEM 4: } f_4^* &= \max f_4(\mathbf{x}) \\ \text{s.t. } & \sum_j x_{ij} = 1, \quad \forall i \\ & \sum_i x_{ij} = 1, \quad \forall j \\ & f_k(\mathbf{x}) \geq p_k f_k^*, \quad k=1,2,3 \\ & x_{ij} \in \{0,1\} \end{aligned}$$

where  $(1-p_k)$ ,  $k = 1, 2, 3$ , are the levels of degradation for Navy needs, career needs, and desires.

#### B. THE ASSIGNMENT PROBLEM WITH SIDE CONSTRAINTS

Clearly, Problem 1 is the standard assignment problem, which can be solved efficiently and naturally yields an integer solution. The remaining problems are, however, complicated by the constraints that ensure the levels of higher objectives are not degraded below the specified amounts. These constraints are commonly referred to as

"side constraints", and their presence destroys the integrality property of the underlying assignment problem.

In a situation where only the optimal value of the objective function is of interest, an assignment problem with side constraints is typically addressed by Lagrangian Relaxation or Subgradient Optimization techniques. See Held, Wolfe, and Crowder (1974), Geoffrion (1974), Pojak (1977), Fisher (1981), and Shor (1985). However, it is well known that Lagrangian Relaxation does not always produce a feasible solution to this type of problem. Since the PO and AO both require feasible assignments in order to effectively perform their tasks, Lagrangian Relaxation would be inappropriate. In the next two chapters, an alternative technique is proposed.

### III. A MULTIOBJECTIVE SCHEME FOR THE NOA PROBLEM

In Chapter II, Problems 2, 3, and 4 all have the following form:

$$\begin{aligned} P: \quad & \max f_{r+1}(\mathbf{x}) \\ & \text{s.t.} \\ & f_k(\mathbf{x}) \geq p_k f_k^*, \quad \forall k=1, 2, \dots, r \\ & \mathbf{x} \in S \end{aligned}$$

where  $r = 1, 2$ , or  $3$  and  $S = \{\mathbf{x}: \sum x_{i,j} = 1 \quad \forall i, \sum x_{i,j} = 1 \quad \forall j$  and  $x_{i,j} \in \{0,1\}\}$ . Applying Lagrangian Relaxation to the above problem would produce the following subproblem to be solved for some  $\mu \geq 0$  ( Geoffrion, 1974):

$$L(\mu) = \max \{f_{r+1}(\mathbf{x}) + \sum_{k=1}^r \mu_k (f_k(\mathbf{x}) - p_k f_k^*) : \mathbf{x} \in S\}$$

However, since  $\mu$  is given and  $p_k f_k^*$  is prespecified, the above problem can be equivalently written as

$$LR: \quad \max \{f_{r+1}(\mathbf{x}) + \sum_{k=1}^r \mu_k f_k(\mathbf{x}) : \mathbf{x} \in S\}$$

Note that LR now has the form of the standard assignment problem and the linear programming relaxation of LR would yield an optimal integer solution. Moreover, the objective function of LR is of the form

$$\sum_{k=1}^{r+1} w_k f_k(\mathbf{x})$$

where  $w_{r+1}$  is set equal to 1. Therefore, the Lagrangian Relaxation approach is equivalent to maximizing the weighted sum of  $r+1$  objective functions, which is exactly how EPANS handles its 14 objectives. As mentioned before, Lagrangian Relaxation does not always produce a feasible solution. If  $\mathbf{x}^*$  is an optimal solution to LR, then it is possible that  $\mathbf{x}^*$  may violate one or more of the side constraints, i.e.,

$$f_k(\mathbf{x}) < p_k f_k^*, \quad \text{for some } k=1,2,\dots,r.$$

This may help explain the erratic behavior of the weighting scheme in which weights corresponding to the intentions of the user produce poor solutions. Based on the above observation, this poor solution may violate the side constraints implicitly articulated in terms of weights by the users. Presented below is a technique which combines this weighting scheme with an appropriate multiobjective programming technique that can be used to produce a good feasible solution to problem P.

#### A. EFFICIENT POINTS AND FRONTIERS

For single objective problems, it is easy to differentiate an optimal solution from a non-optimal solution by simply comparing their objective function values. When a problem has more than one objective function, vectors of function values must be compared; an impossible task since there exists no complete ordering in



vector space with dimension higher than one. Practically speaking, when there is more than one objective function, the objectives usually represent conflicting points of view or goals. An improvement in one of the objectives usually means a degradation in one or more of the others. Unless there exists an "economic utility" function combining the vector of objective function values into a single number, it will be difficult to compare two feasible solutions when more than one objective is present. To deal with this difficulty, the concept of an efficient solution is proposed. Instead of providing the user with optimal solutions, he or she will be provided with a set of efficient solutions.

Consider the following multiobjective programming problem:

$$\text{MP:} \quad \text{vmax} \quad \left\{ \begin{array}{c} f_1(\mathbf{x}) \\ f_2(\mathbf{x}) \\ . \\ . \\ f_l(\mathbf{x}) \end{array} \right\} \quad \text{subject to} \quad \mathbf{x} \in S$$

where 'vmax' is an abbreviation for vector maximization and  $f_k(\mathbf{x})$  is a linear function in  $\mathbf{x}$  for all  $k$ . The next two definitions define the concept of efficiency (Steuer, 1986, p.148):

**Definition 1:** Let  $\mathbf{x}$  and  $\mathbf{y}$  be two feasible solutions to problem MP. Then  $\mathbf{x}$  dominates  $\mathbf{y}$  if

$$\begin{aligned} f_k(\mathbf{x}) &\geq f_k(\mathbf{y}), \text{ for } k=1,2,\dots,r, \text{ and} \\ f_k(\mathbf{x}) &> f_k(\mathbf{y}), \text{ for at least one } k \in \{1,2,\dots,r\}. \end{aligned}$$

**Definition 2:** A feasible solution  $\mathbf{x}$  is efficient or noninferior if there exists no solution  $\mathbf{y} \in S$  that dominates  $\mathbf{x}$ .

To illustrate the concepts of domination and efficiency of feasible solutions, consider the following two objective assignment problem (White, 1984):

$$\begin{aligned} \text{vmax} \quad & \begin{cases} f_1(\mathbf{x}) = \sum_{i,j} C_{ij} x_{ij} \\ f_2(\mathbf{x}) = \sum_{i,j} D_{ij} x_{ij} \end{cases} \\ \text{s.t.} \quad & \sum_{j=1}^3 x_{ij} = 1, \quad i=1,2,3 \\ & \sum_{i=1}^3 x_{ij} = 1, \quad j=1,2,3 \\ & x_{ij} \in \{0,1\} \end{aligned}$$

where

$$C = \begin{bmatrix} 8 & 9 & 10 \\ 9 & 8 & 1 \\ 10 & 1 & 8 \end{bmatrix}, \quad D = \begin{bmatrix} 10 & 7 & 8 \\ 10 & 10 & 1 \\ 8 & 1 & 10 \end{bmatrix}$$

There are  $3!$  or 6 feasible solutions to this example; they are

- (1)  $x_{11}^1 = x_{22}^1 = x_{33}^1 = 1, x_{ij}^1 = 0$  otherwise; and  
 $f_1(\mathbf{x}^1) = 24, f_2(\mathbf{x}^1) = 30$
- (2)  $x_{11}^2 = x_{23}^2 = x_{32}^2 = 1, x_{ij}^2 = 0$  otherwise; and  
 $f_1(\mathbf{x}^2) = 10, f_2(\mathbf{x}^2) = 12$
- (3)  $x_{12}^3 = x_{21}^3 = x_{33}^3 = 1, x_{ij}^3 = 0$  otherwise; and  
 $f_1(\mathbf{x}^3) = 26, f_2(\mathbf{x}^3) = 27$

- (4)  $x_{12}^4 = x_{23}^4 = x_{31}^4 = 1$ ,  $x_{13}^4 = 0$  otherwise; and  
 $f_1(x^4) = 20$ ,  $f_2(x^4) = 16$
- (5)  $x_{13}^5 = x_{21}^5 = x_{32}^5 = 1$ ,  $x_{12}^5 = 0$  otherwise; and  
 $f_1(x^5) = 20$ ,  $f_2(x^5) = 19$
- (6)  $x_{13}^6 = x_{22}^6 = x_{31}^6 = 1$ ,  $x_{12}^6 = 0$  otherwise; and  
 $f_1(x^6) = 28$ ,  $f_2(x^6) = 26$

Figure 2 displays the values of the two objective functions for the six solutions. In general, the graphical displays with axes representing the values of the objective functions are called the objective space. Thus, Figure 2 actually shows all the feasible solutions of the example problem in the objective space.

From this figure, it is clear that

- $x^1$  dominates  $x^2$ ,  $x^4$ , and  $x^5$
- $x^3$  dominates  $x^2$ ,  $x^4$ , and  $x^5$
- $x^4$  dominates  $x^2$
- $x^5$  dominates  $x^2$  and  $x^4$
- $x^6$  dominates  $x^2$ ,  $x^4$ , and  $x^5$ .

Since  $x^1$ ,  $x^3$ , and  $x^6$  are not dominated by any other favorable solution, they must be efficient.

To introduce the concept of efficient frontiers, consider the linear programming relaxation of problem MP in which  $x_{1j}$  is allowed to vary continuously in the interval between 0 and 1, i.e. replace S with its convex hull as the feasible region. For the example problem, this corresponds to replacing the constraint  $x_{1j} \in \{0,1\}$  with  $0 \leq x_{1j} \leq 1$ . Under this relaxation, the feasible region in the objective

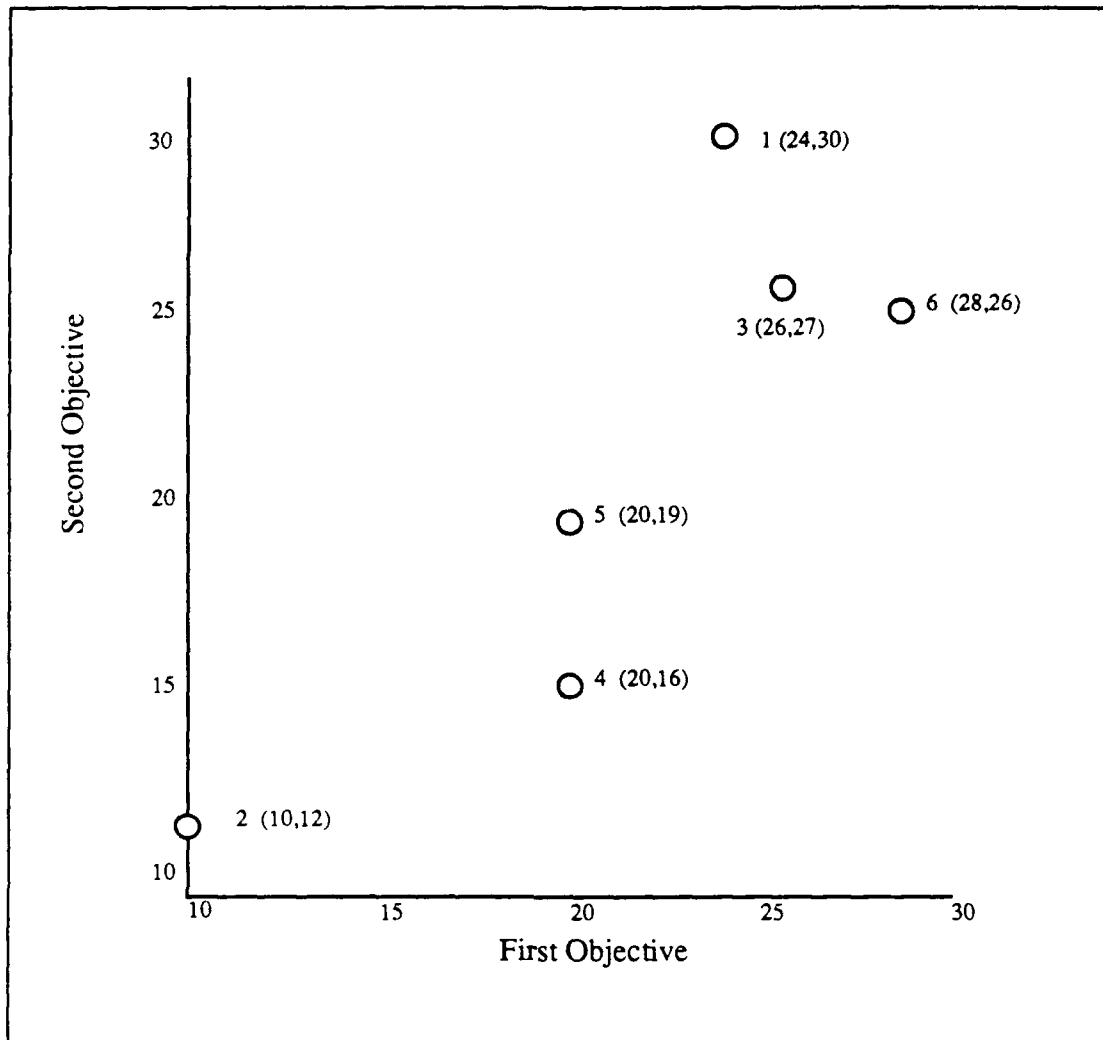


Figure 2: The feasible region in the objective space.

space for the relaxed problem is simply the convex hull of all of the feasible solutions to the original problem MP (see Figure 3). Note that the solutions along the line joining  $x^1$  and  $x^6$  constitute the entire set of efficient solutions. It is this set of efficient solutions for the relaxed problem that defines the efficient frontier for the original problem. Formally, the efficient frontier is defined as follows

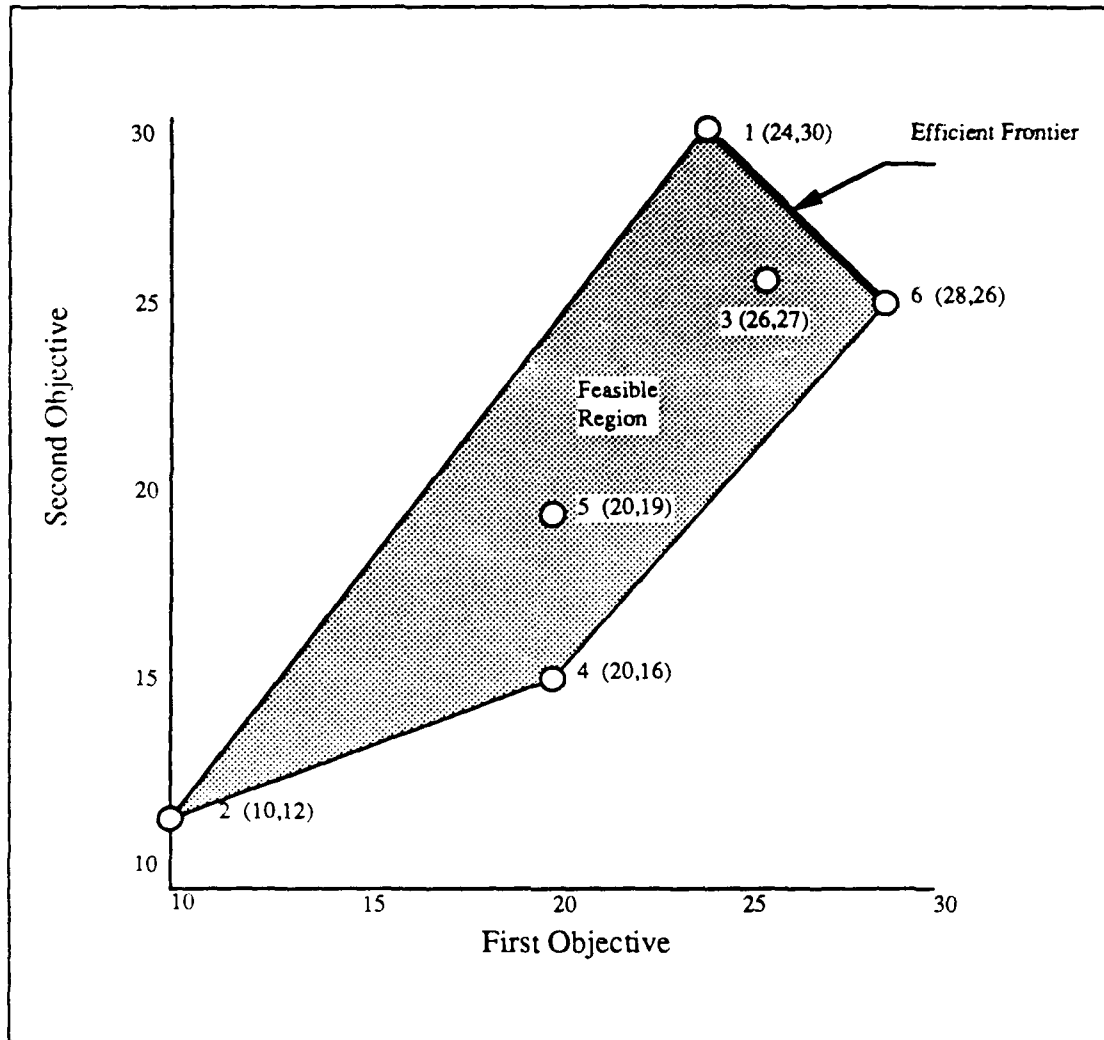


Figure 3: The feasible region in the objective space for the relaxed problem.

**Definition 3:** The efficient frontier for problem MP is the set of efficient solutions for the linear programming relaxation of problem MP.

Observe that the efficient frontier of the example problem does not contain all the efficient solutions to the problem. In particular,  $x^3$  does not belong to the efficient frontier. In fact, it is well known (see, e.g. White (1984) and Bazaara and Shetty (1979)) that Lagrangian Relaxation

and the weighted sum scheme (using nonnegative weights) only produce extreme efficient solutions which lie on the frontier, e.g.  $x^1$  and  $x^6$ . For the NOA problem, it is anticipated that the frontier will be densely populated by extreme efficient solutions such that those efficient solutions not on the frontier can be ignored without greatly affecting the assignments. Since the coefficients for Navy needs, career needs, and desires must be artificially constructed, it also seems reasonable to only consider the efficient frontiers; otherwise a much more complicated and time consuming procedure must be considered.

#### B. AN ALGORITHM FOR SOLVING AN ASSIGNMENT PROBLEM WITH SIDE CONSTRAINTS

In this section, an algorithm is presented to solve problem P. The algorithm is a modification of the Noninferior Set Estimation (NISE) method (Cohon (1978) and Solanki and Cohon (1989)) for approximating the efficient frontier of a multiobjective programming problem. This method is based on the following well known result

**Theorem:** If  $x^*$  is a solution to the following problem

$$WP: \quad \max \left\{ \sum_{k=1}^r w_k f_k(x) : x \in S \right\}$$

where  $w_k > 0$  for all  $k=1,2,\dots,r$  and  $S$  is the feasible region of the NOA, then  $x^*$  is an efficient solution to the multiobjective programming problem MP.

If  $w_k = 0$  for some  $k$ , problem WP may admit optimal solutions and some solutions of which may be dominated.

In estimating the efficient frontier of problems with  $k$  objectives, the NISE method constructs a hyperplane passing through  $k$  efficient points in the objective space. The weights  $w_k$  are then constructed from the "slopes" of this hyperplane and problem WP is solved, thereby obtaining another (extreme) efficient solution. The result of solving problem WP is akin to "pushing" the hyperplane to the efficient frontier of the feasible region. Figure 4 depicts this process for a biobjective problem.

The key to the efficiency of the NISE method is the selection of the  $k$  efficient points from which to construct the hyperplane. The algorithm below is based on this same principle; however, the selection rule for the  $k$  efficient points is different. In the NISE method, the desired result is an estimation of the entire efficient frontier. For the NOA problem, we are interested in only the portion of the frontier which contains a solution to problem P (Figure 5).

Below, a modified version of the NISE algorithm is formally presented. To simplify this presentation, problems in the form of WP are assumed to have unique solutions which must therefore be efficient.

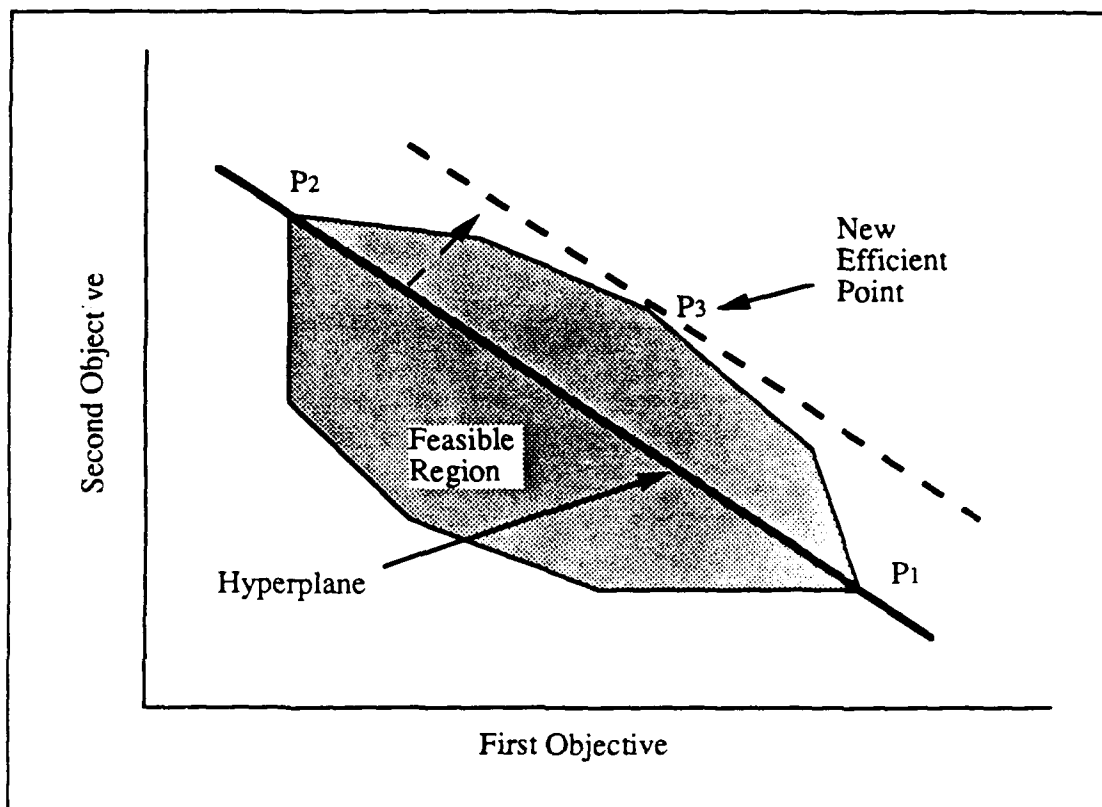


Figure 4: Finding an efficient solution to the NOA problem in the objective space.

#### The Modified NISE Algorithm

Step 0: Let  $x^k$ , where  $k = 1, 2, \dots, r$ , be the solution to

$$\max \{f_k(x) : x \in S\}$$

Set  $I = \{x^1, x^2, \dots, x^r\}$  and construct an  $r$ -dimensional hyperplane through the points in  $I$  and obtain weights (slopes)  $W = (w^1, w^2, \dots, w^r)$ . Go to Step 1.



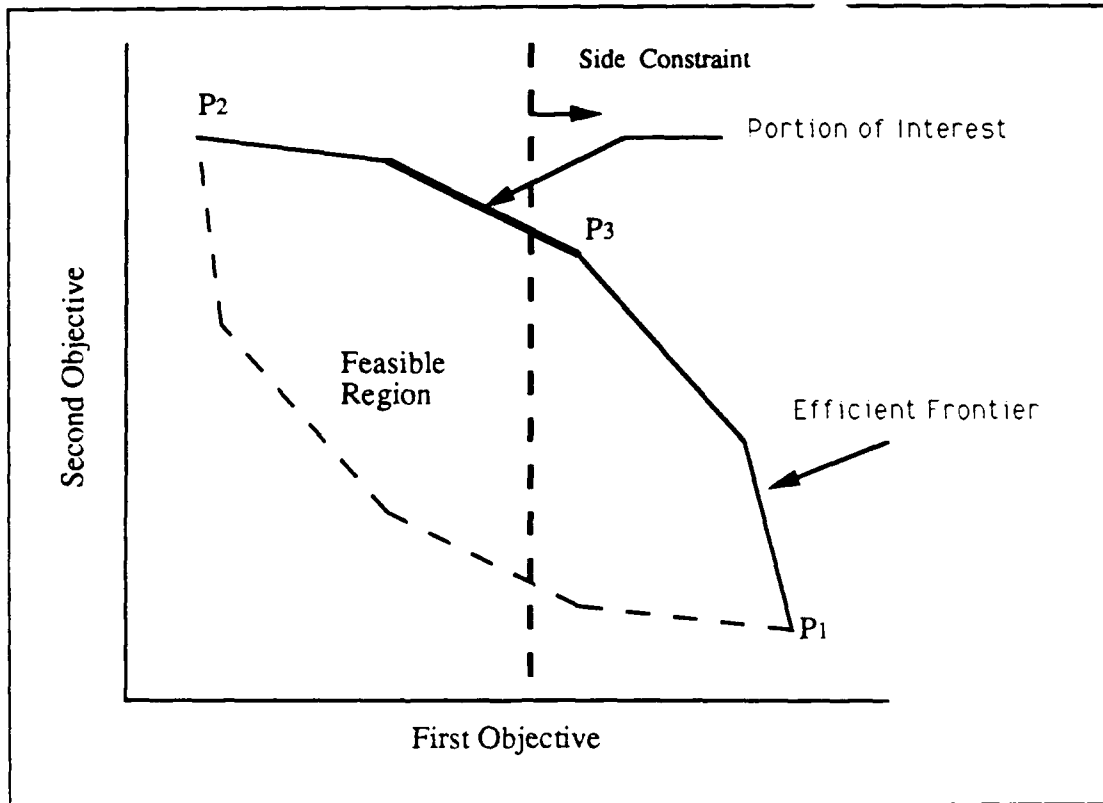


Figure 5: The portion of interest of the efficient frontier for solving the NOA problem.

Step 1: Let  $y$  solve

$$\max \left\{ \sum_{k=1}^r w_k f_k(\mathbf{x}) : \mathbf{x} \in S \right\}$$

If  $y \in I$ , then stop. One of the  $\mathbf{x}^i \in I$  is a solution.

Step 2: Define the set  $J^k = \{\mathbf{x}^1, \mathbf{x}^2, \dots, \mathbf{x}^{k-1}, y, \mathbf{x}^{k+1}, \dots, \mathbf{x}^r\}$  for  $k = 1, 2, \dots, r$ . Then construct  $r$  hyperplanes passing through points in  $J^k$ ,  $k = 1, 2, \dots, r$ , and let  $Z^k = (z_1^k, z_2^k, \dots, z_r^k)$  be the weights (slopes) from hyperplane  $k$ . If some component of  $Z^k$  is negative, replace  $Z^k$  with  $W + \alpha(Z^k - W)$ , where

$$\alpha = \max \{ \lambda : W + \lambda(Z^k - W) \geq 0 \text{ and } \lambda \geq 0 \}$$

Step 3: Let

$$d = \begin{bmatrix} f_1(y) - \rho_1 f_1^* \\ f_2(y) - \rho_2 f_2^* \\ \vdots \\ f_r(y) - \rho_r f_r^* \end{bmatrix}$$

and determine  $k^*$  which satisfies

$$\frac{d^T(Z^{k^*} - W)}{\|d\| \|Z^{k^*} - W\|} = \max \left\{ \frac{d^T(Z^k - W)}{\|d\| \|Z^k - W\|} : k = 1, 2, \dots, r \right\}$$

Then replace  $I$  with  $J^{k^*}$  and set  $W = Z^{k^*}$ . Go to Step 1.

In Step 0, the set  $I$  represents the first set of  $r$  efficient points. Choosing  $x^k$  as the maximizer of the  $k^{\text{th}}$  objective ensures that the portion of the efficient frontier containing the solution is not excluded. Step 1 then "pushes" the current hyperplane out to the efficient frontier, thereby producing a new efficient solution,  $y$ . Using  $y$  and  $k$  other efficient points in the set  $I$ , Step 2 forms  $r$  hyperplanes each of which produces a set of  $r$  vectors of weights,  $Z^k$ ,  $k = 1, 2, \dots, r$ . For a given combination of efficient points, i.e.  $x^1, x^2, \dots, x^{k-1}, y, x^{k+1}, \dots, x^r$ , the vector weights,  $Z$ , can be obtained by solving the following linear simultaneous equations:

$$\begin{array}{ccccccc}
w_1 f_1(x^1) & + & w_2 f_2(x^1) & + & \dots & + & w_r f_r(x^1) & = & F^* \\
. & & . & & & & . & & . \\
. & & . & & & & . & & . \\
w_1 f_1(x^{k-1}) & + & w_2 f_2(x^{k-1}) & + & \dots & + & w_r f_r(x^{k-1}) & = & F^* \\
w_1 f_1(y) & + & w_2 f_2(y) & + & \dots & + & w_r f_r(y) & = & F^* \\
w_1 f_1(x^{k+1}) & + & w_2 f_2(x^{k+1}) & + & \dots & + & w_r f_r(x^{k+1}) & = & F^* \\
. & & . & & & & . & & . \\
. & & . & & & & . & & . \\
w_1 f_1(x^r) & + & w_2 f_2(x^r) & + & \dots & + & w_r f_r(x^r) & = & F^*
\end{array}$$

where  $F^*$  is the optimal objective value obtained in Step 1. However, these equations may produce a vector  $Z$  with some negative components, in which case  $Z$  is replaced by another vector  $Z' = W + \alpha(Z-W)$ . The value of  $\alpha$  is chosen so that  $Z'$  is as far away from  $W$  as possible while remaining nonnegative for all values.

Finally, in Step 3, the vector  $d$  is a subgradient of the Lagrangian dual function and the weight vector  $z^{k*}$  which yields the maximum inner product is selected to be used as the new weight vector in Step 1. At this point, the algorithm begins a new iteration. Note that the subgradient of the Lagrangian dual function is utilized in the process of selecting a new hyperplane because the slope of the portion of the frontier which contains the solution is exactly the optimal solution to the Lagrangian dual of problem  $P$ , i.e.,

$$\min \{L(\mu): \mu \geq 0\}$$

where  $L(\mu)$  is as defined at the beginning of this chapter.

### C. A BIOBJECTIVE EXAMPLE PROBLEM

To illustrate the above algorithm, consider problem P with  $r = 1$ , i.e.,

$$\begin{aligned} \max & f_2(\mathbf{x}) \\ \text{s.t.} & \\ & f_1(\mathbf{x}) \geq p_1 f_1^* \\ & \mathbf{x} \in S \end{aligned}$$

When considered as a biobjective problem, the problem is assumed to have efficient solutions denoted  $P_1, P_2, \dots, P_5$ . Figure 6 depicts these five solutions in the objective space. The vertical line in the figure represents the side constraint,  $p_1 f_1^*$ . It is clear from this figure that  $P_4$  is the desired solution.

The algorithm starts by separately maximizing the two objectives,  $f_1(\mathbf{x})$  and  $f_2(\mathbf{x})$ , to obtain  $P_1$  and  $P_2$ , respectively. So, the initial set I consists of the points  $P_1$  and  $P_2$ . Drawing a line through  $P_1$  and  $P_2$  produces a hyperplane with slope  $-w_1/w_2$ , where  $w_1$  and  $w_2$  are the weights. Then, solving the weighted problem in Step 1, i.e. "pushing" the line out to the frontier, a new efficient solution,  $P_3$ , is obtained (Figure 7).

In Step 2, there are two sets of weights: one set corresponds to the line joining  $P_1$  and  $P_3$  and the other set to the line joining  $P_2$  and  $P_3$ . Note that both sets of weights are nonnegative since both lines have negative

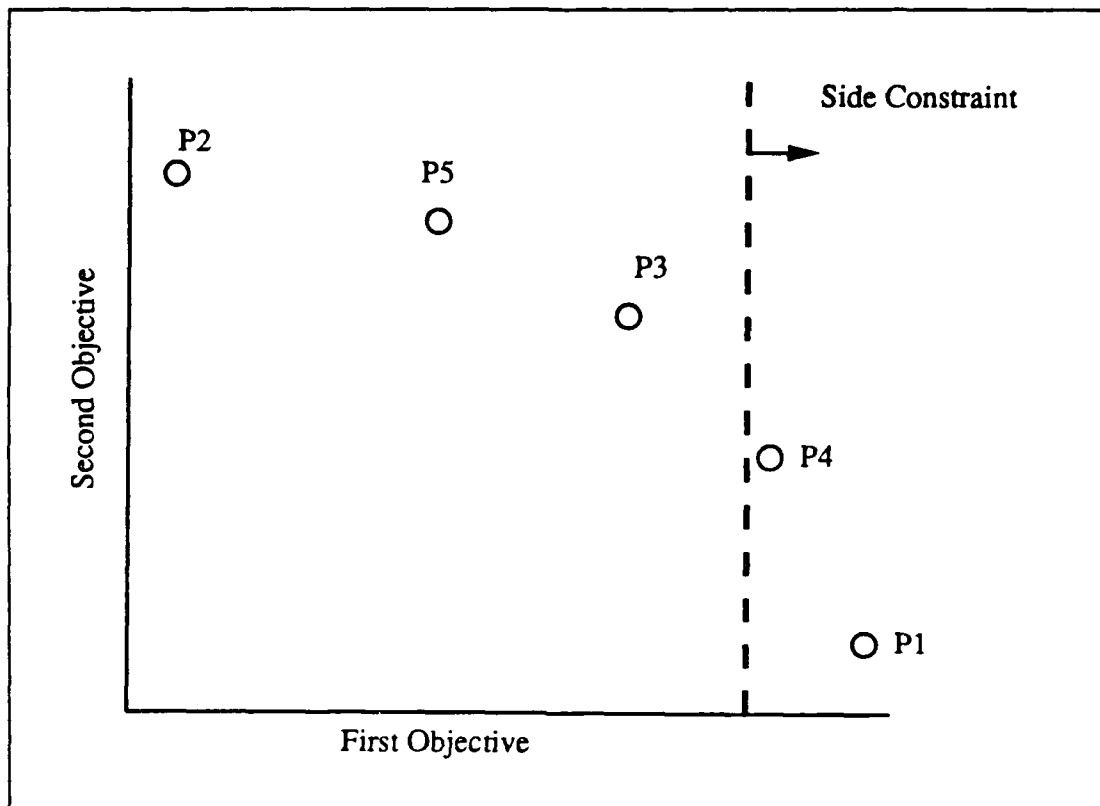


Figure 6: Efficient solutions in objective space to an example biobjective problem.

slopes. Using the criterion in Step 2, the line joining  $P_1$  and  $P_3$  is selected, i.e.  $I = \{P_1 \text{ and } P_3\}$ , at which point the algorithm returns to Step 1 and a new iteration begins. The problem in Step 1 is solved with the new weights from the slope of the line between  $P_1$  and  $P_3$ , producing a new efficient point,  $P_4$  (Figure 8). Step 2 then produces two new sets of weights: one set corresponding to the line joining  $P_1$  and  $P_4$  and the other set to the line joining  $P_3$  and  $P_4$ . The set of weights corresponding to the line joining  $P_3$  and  $P_4$  is selected and set  $I$  now contains  $P_3$  and  $P_4$ . The algorithm then begins a new iteration at Step 1.

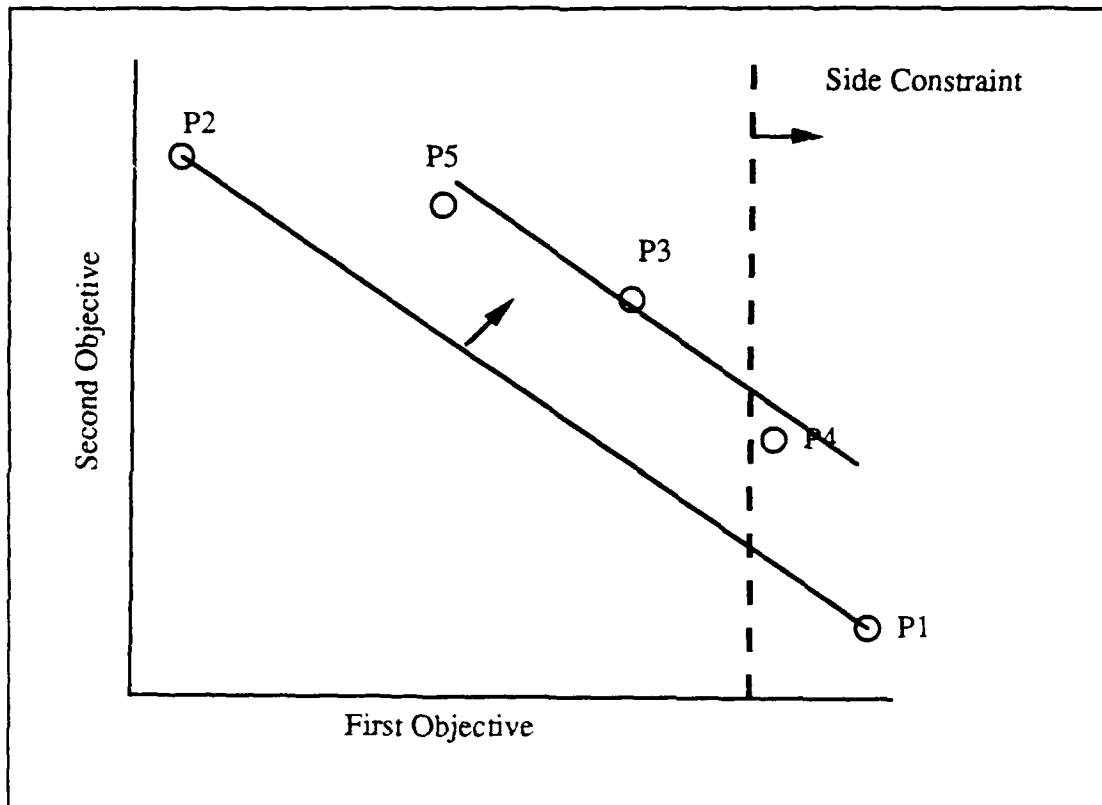


Figure 7: Obtaining point  $P_3$  using the modified NISE algorithm.

The weighted problem is solved producing the optimal solution  $P_4$ , which is already a member of set  $I$ , so the algorithm stops and  $P_4$  is the desired solution (Figure 9).

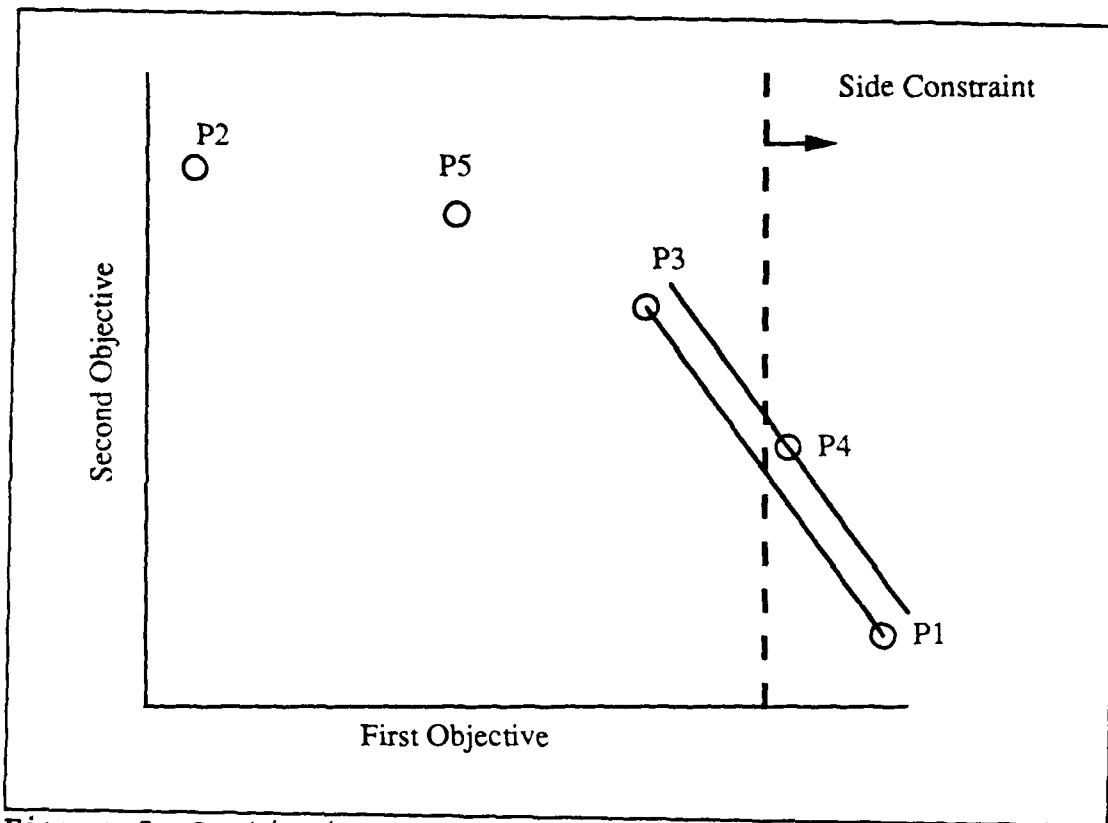


Figure 8: Continuing the modified NISE procedure to find  $P_4$ .

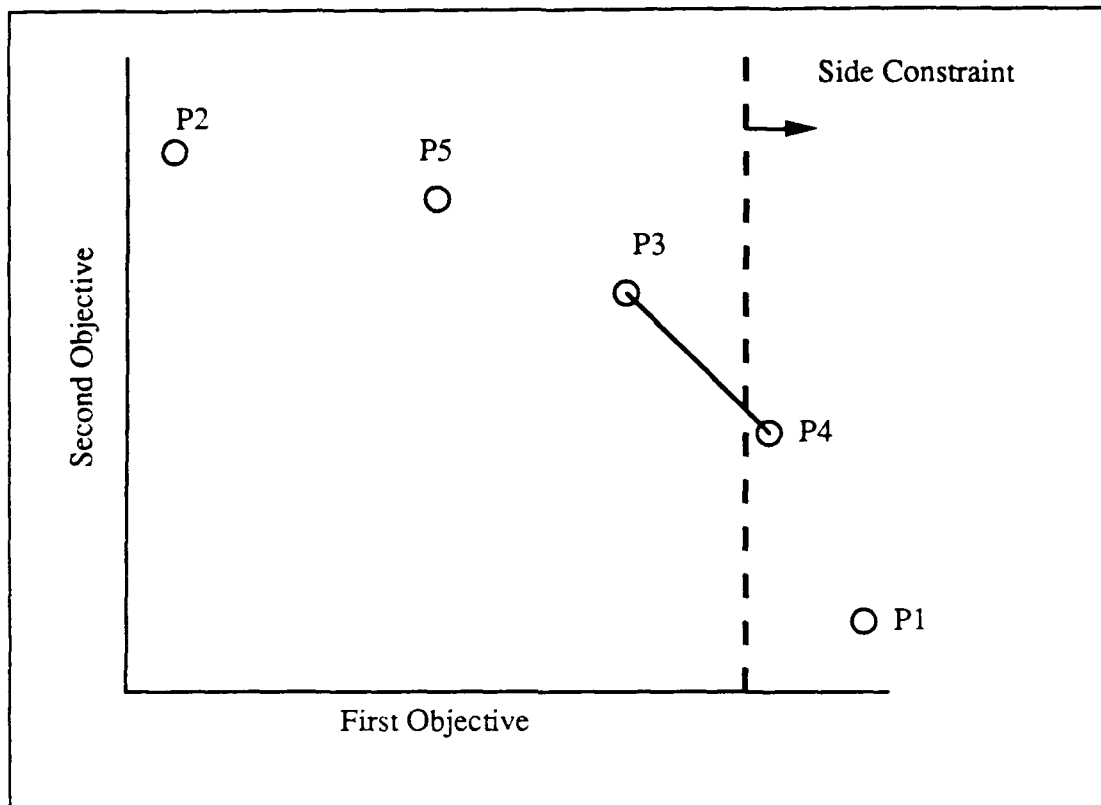


Figure 9: Determining the desired solution point  $P_4$ .



#### IV. EXAMPLE PROBLEM

This chapter applies the modified NISE method to a sample assignment problem involving junior Naval officers with an aviation warfare designation and a specific AQD. Because of the Privacy Act, part of the data for the problem is artificially constructed. However, every attempt is made to ensure that the sample problem is representative of the actual Navy officer assignment problem.

The main objective of the chapter is to demonstrate the validity of the method as well as to indicate the type of information which can be provided to the user. It should be stressed that the information provided is intended to be used as an aid in decision making and not as a decision.

The following are assumptions utilized in constructing the database for the example problem. Although these assumptions simplify the data construction process, the resulting problem still contains all aspects of the real assignment problem.

- There are four objectives to be considered: the "Triad of Detailing" and the cost. The hierarchy of these four objectives is as discussed in Chapter I.
- The decision maker will only accept noninferior objective values.
- Assignments requiring special handling are removed from the problem. Examples are sea-to-sea requests or officers that have failed for promotion.
- The Navy needs objective depends only on warfare

designation, AQD, subspecialty codes, sea-shore rotation, rank, PRD, and billet priority.

- Officers designated 131X are eligible for billets designated 131X or 130X, officers designated 132X are eligible for billets designated 132X or 130X.
- The Navy needs objective cannot be zero for any assignment.
- The cost of an assignment is proportionally related to the distance from an officers present billet to the proposed billet.

#### A. DATABASE DESCRIPTION

In the actual assignment process, the AO and PO obtain information concerning the officers and billets from the Officer Assignment Information System (OAIS) (NMPC-47, 1989, pp. 1-8). The OAIS is an automated data processing system which maintains files containing information regarding the officers and billets such as the Officer Master File (OMF), the Billet File (BF), the Fitness Report File (FITREP), and the Duty Preference File (DPF). However, because of the Privacy Act, NPRDC, the main source of data for this study, can only provide a portion of data from the OMF and the BF files.

The OMF file contains information such as rank, warfare designation, AQD, PRD, subspecialties, and previous duty stations for active Naval officer personnel. Fifty one officers meeting the following criteria were selected to form the database for the sample problem:

- Aviation warfare designation of 1310, 1315, 1320, or

1325 (131X is a pilot and 132X is a naval flight officer; XXX5 is a reserve officer on active duty)

- Rank of Lieutenant Junior Grade (LTJG), Lieutenant (LT), or Lieutenant Commander (LCDR).
- PRD between 8811 and 8907.
- AQD of DK2 (EP-3 aircraft qualified).

Additional information such as current billet location, subspecialty codes, and present duty type (sea or shore) were also obtained for these selected officers. Based on the above criteria, these fifty one officers have the same AO since they have the same AQD.

A sample of billets with location, required subspecialty code, and duty type was also extracted from the BF files. All billets meet the following criteria:

- Aviation warfare designator code of 131X, 132X, or 130X.
- Rank required of LTJG, LT, or LCDR.
- PRD between 8811 and 8907 (the billet PRD was obtained by cross-referencing the incumbent officer's PRD from the OMF with the BF; the BF does not contain PRD).
- AQD of DK2.

To ensure that the resulting assignment is feasible, additional shore duty billets requiring no specific AQD were added. The number of billets for the problem is fifty eight.

The Navy needs satisfied by assigning officer i to billet j is computed using the following formula:

$$C_{ij} = SEA_{ij} * SKILL_{ij} * PRIORITY_j * (RANK_{ij} + PRD_{ij} + 1)$$

where

- 1)  $SEA_{ij}$  represents the Navy sea-shore rotation policy. If an officer is presently at a sea billet, then

$$\begin{aligned} SEA_{ij} &= 1 && \text{if billet } j \text{ is a shore billet} \\ &= 0 && \text{if billet } j \text{ is a sea billet.} \end{aligned}$$

- 2)  $SKILL_{ij}$  represents how well officer  $i$  qualifies for billet  $j$ . In particular,

$$\begin{aligned} SKILL_{ij} &= 1 && \text{if officer } i \text{ and billet } j \text{ have the} \\ &&& \text{same warfare designation.} \\ &= 2 && \text{if officer } i \text{ and billet } j \text{ have the same} \\ &&& \text{warfare designation and AQD code.} \\ &= 4 && \text{if officer } i \text{ and billet } j \text{ have the same} \\ &&& \text{warfare designation, AQD code, and} \\ &&& \text{subspecialty code.} \\ &= 0 && \text{otherwise.} \end{aligned}$$

- 3)  $PRIORITY_j$  represents the importance of filling the billet. The priority level which ranges from 1 to 3 is assigned to a billet based on the Chief of Naval Operations (CNO) manning requirement, skill scarcity, and other factors as specified by the PO.
- 4)  $RANK_{ij}$  has the value of one if the billet  $j$  requires the same or one rank higher than the rank of the officer  $i$  being proposed for the billets. In practice, assigning values to  $RANK_{ij}$  should reflect the current NMPC policy. Both the AO and PO must evaluate the values of  $RANK_{ij}$  carefully in order to ensure that the resulting assignments satisfy NMPC policy.
- 5)  $PRD_{ij}$  has the value of one if the PRD of officer  $i$  is within three months of vacancy date for billet  $j$ . The length of three months may be adjusted on a case by case basis to suit the needs of the AO and the PO in different circumstances.

Table 2 tabulates the minimum and the maximum values for each of the factors in the Navy needs equation. It should be noted that the computer program for the NOA problem disallows the assignment of officer  $i$  to billet  $j$  if  $C_{ij}$  for

that assignment is zero. So, by setting  $SEA_{ij}$ , and/or  $SKILL_{ij}$ , to zero, the AO and the PO can guarantee that (1) each officer will not be assigned to sea duty on two consecutive rotations and (2) each officer has the appropriate skill for his or her new billet.

TABLE 1. NAVY NEEDS OBJECTIVE FUNCTION.

VARIABLE NAME	MINIMUM VALUE	MAXIMUM VALUE
$Skill_{ij}$	0	4
$Sea_{ij}$	0	1
$Priority_{ij}$	1	3
$PRD_{ij}$	0	1
$Rank_{ij}$	0	1
$C_{ij}$	0	36

The cost of assigning officer  $i$  to billet  $j$ ,  $F_{ij}$ , is computed from the distance between the current location of officer  $i$  and the location of billet  $j$ . It is assumed for the sample problem that the transportation cost is one cent per mile. Based on the obtained data,  $F_{ij}$  varies from \$0 to \$97. In practice, there are many factors which should be included in the calculation of  $F_{ij}$  and it would be beyond the scope of this thesis to list and discuss them. The values for the remaining two objectives, career ( $D_{ij}$ ) and desires ( $E_{ij}$ ), were randomly generated within the ranges listed in Table 2.

TABLE 2. CAREER NEEDS, DESIRES,  
AND COST OBJECTIVE FUNCTIONS.

VARIABLE NAME	MINIMUM VALUE	MAXIMUM VALUE
$D_{1j}$	0	2
$E_{1j}$	0	3
$F_{1j}$	0	97

#### B. IMPLEMENTATION OF THE MODIFIED NISE ALGORITHM

The modified NISE algorithm has been implemented in a FORTRAN program that determines the set of assignments that best satisfy the user's objectives. The program uses an out-of-kilter network algorithm as a solver for the purpose of solving the example problem (Woolsey, 1975, pp.115-117). Much more efficient network solvers, such as GNET, exist and can be used to increase the speed of the algorithm. Gauss-Jordan Elimination is used to determine the weights (slopes) of the combined objective function for Step 0 of the modified NISE algorithm.

As input to the program, the user must provide the following:

- The database described in the previous section;
- The number of objectives,  $k$ ;
- The lower bounds for  $k-1$  objectives that are higher in the hierarchy,  $p_k f_k^*$ .

The output from the program will be  $k$  efficient solutions that best meet the lower bounds. The user will pick a solution and view the set of assignments associated

with it, or proceed to a problem with higher objectives.

For the example problem, the user begins with a biobjective assignment problem. The first and second objectives in the hierarchy, Navy needs and career needs, are maximized to produce the efficient points in objective space,  $P_1$  and  $P_2$ . The values for these two objectives, as well as the two other objectives lower in the hierarchy, are displayed in Table 3.

The user wishes to increase the career needs objective value, so a 5% reduction in Navy needs is allowed. The lower bound of this biobjective problem is 95% of 910, the maximum value of the Navy needs objective function, or 865.

TABLE 3. OBJECTIVE VALUES FOR THE TWO OBJECTIVE PROBLEM.

Objective	Navy Needs	Career	Desires	Cost
$P_1$	910	82	74	1625
$P_2$	823	102	56	2108
$P_5$	865	99	64	1984

The modified NISE program traverses the efficient frontier and produces the efficient point  $P_5$  after five iterations (Table 3). This point meets the lower bound of the Navy needs objective, 865, exactly. As can be seen from Table 3, the career needs objective value increased from that found at point  $P_1$ .

If the user is satisfied with the resulting solution,

$P_5$ , the program will produce the associated set of assignments. Based on the equations described earlier, Navy needs can take on several different values as displayed in the first row of Table 4. The other three rows correspond to the number of assignments which achieve each of the Navy needs values for different assignment solutions;  $P_1$ ,  $P_2$ , and  $P_5$ . The solution  $P_1$  has more assignments achieving higher values of Navy needs since only Navy needs is being maximized. The solution  $P_2$  is where career needs is maximized without regard to Navy needs. So, it is only logical that  $P_2$  does not have as many assignments achieving higher Navy needs values. In fact,  $P_2$  has three assignments with Navy needs of 4 or lower while  $P_1$  has none. Similar results can also be obtained when comparing  $P_1$  and  $P_5$ . In the same manner, Table 5 illustrates the distribution of assignment values for career needs which can have only three different values: 2 (high), 1 (medium), and 0 (low). These different assignments produce total objective scores that meet the lower bound of Navy needs and maximize the career needs. To manually determine these assignments would require the consideration of  $58!/(58-51!)$ , or  $4.66 \times 10^4$  combinations.



TABLE 4. FREQUENCY OF NAVY NEEDS ASSIGNMENT VALUES.

Solutions	Values for Navy needs									
	36	24	18	16	12	8	6	4	2	1
$P_1$	9	7	2	10	11	9	3	0	0	0
$P_2$	7	6	3	11	8	8	4	3	0	0
$P_5$	8	8	2	10	9	6	3	3	1	1

TABLE 5. FREQUENCY OF CAREER NEEDS ASSIGNMENT VALUES.

Solutions	Values for career needs		
	2 (high)	1 (medium)	0 (low)
$P_1$	35	12	4
$P_2$	51	0	0
$P_5$	48	3	0

Assuming that the user wishes to proceed further, a three objective problem is attempted. The program begins and produces the objective values found in Table 6. Note that the values of the second, third, and fourth objective functions for  $P_1$  are different than those found in Table 3. The point  $P_2$  also has different values for the objective functions, save the second objective. This is because the consideration of the third objective makes the efficient frontier a surface, vice the curve found in the biobjective problem.

Deciding upon the amount of degradation for each objective value, say 5% for both, the lower bounds become 865 and 94 for Navy needs and career needs, respectively.

After 18 iterations of the modified NISE procedure, the point  $P_{1-}$  is produced as an efficient solution. Table 6 shows that for  $P_{1-}$ , the Navy needs value is just above the lower bound, while the career needs value is exactly met. The point closest to the bounds that maximizes the third objective is obtained.

TABLE 6. OBJECTIVE VALUES FOR THE THREE OBJECTIVE PROBLEM.

Objective	Navy Needs	Career	Desires	Cost
$P_1$	910	80	90	1608
$P_2$	818	102	85	1892
$P_3$	736	57	153	1707
$P_{1-}$	866	94	105	1698

Proceeding in the same fashion for a four objective assignment problem, setting the lower bounds for Navy needs, career needs, and desires at 95% (865, 94, 100) results in the point  $P_{1-}$  being accepted as the final solution to the NOA problem. This is the same solution as the three objective problem, meaning there does not exist a more cost effective solution that meets the bounds specified for the objectives higher in the hierarchy.

## V. CONCLUSION

In this thesis, the problem of assigning naval officers to billets is viewed as an optimization problem with several objectives. To reflect the existing policy at Naval Military Personnel Command (NMPC), these objectives, which include Navy needs, career needs, desires, and cost, are treated in a hierarchical order. In particular, each objective is optimized in the decreasing hierarchical order of importance. While optimizing one objective, the other objectives higher in the hierarchy are constrained above specified values or levels. These values represent a fraction of the maximum attainable values of the objectives (e.g. 95%).

It is well known that the standard assignment problem is a network flow model for which there exists a highly efficient algorithm. However, when the constraints ensuring the minimum levels of objectives higher in the hierarchy are added, the network structure can no longer be exploited and the solution is not guaranteed to have integer values. Chapter III introduces an approach which is a modification of the Noninferior Set Estimation (NISE) technique originally developed by Cohon (1978). This approach both utilizes the network structure and guarantees integer solutions.

The main advantage of the modified NISE method is the fact that it does not require user supplied weights for the objectives as in the weighted sum approach taken by the Enlisted Personnel Allocation and Nomination System (EPANS) of Liang and Thompson (1987). It is well documented that the weighted sum approach produces unpredictable results at times, in that an increase in the weight applied to an objective does not always result in an increase in the value of the corresponding objective function. Moreover, the modified NISE method also allows the user to visualize the assignment process, thereby obtaining a better understanding of the trade-offs between conflicting objectives.

The modified NISE method, incorporated in a FORTRAN program, was applied to a sample Navy officer assignment problem. The sample problem was representative of actual assignment problems faced by the Assignment and Placement officers at NMPC. The results from this example, which were described in Chapter IV, clearly indicate the potential of the modified NISE technique in solving the assignment problem with hierarchical objectives. It was also demonstrated that the information provided by the program can be quite useful as an aid in decision making. Since no model can capture all aspects of the problem, the results from this approach should not be used as the sole criteria for a decision involving the assignment of officers to billets.

Below is a list of possible enhancements and extensions to the modified NISE technique.

- 1) When only three objectives are present, it is possible to graphically illustrate to the user the procedure using three dimensional computer graphics. This can be achieved in the four objective Navy officer assignment model by combining two related objectives such as career needs and desires.
- 2) To achieve better user acceptance, the construction of each objective function should be further studied, perhaps statistically.
- 3) As mentioned in Chapter IV, a better network solver can be incorporated to accelerate the computational time of this approach.
- 4) Further applications to a more general network flow problem can be investigated.
- 5) Other theoretical properties in addition to those discussed in this thesis can be explored.

## LIST OF REFERENCES

- Bazararaa, Mokhtar S. and Shetty, C. M., Nonlinear Programming: Theory and Algorithms, John Wiley & Sons, 1979.
- Cohon, Jared L., Multiobjective Programming and Planning, Academic Press, 1978.
- Fisher, Marshall L., "An Applications Oriented Guide to Lagrangian Relaxation", Interfaces, Vol. 15, No. 2 March-April 1985.
- Geoffrion, A. M., "Lagrangian Relaxation for Integer Programming", Mathematical Programming Study 2, 1974.
- Held, M. D., Wolfe, P. and Crowder, H.P., "Validation of Subgradient Optimization", Mathematical Programming, Vol. 3, 1974.
- Liang, T.T., and Thompson, T.J., "Applications and Implementation: A Large-scale Personnel Assignment Model for the Navy", Decision Sciences, Vol. 18, No. 20, Spring 1987.
- NMPC-4 INSTRUCTION 5400.1G, Officer Distribution Manual, Department of the Navy, December 1982.
- NMPC-47, Navy Military Personnel Distribution System User's Manual, Department of the Navy, January 1989.
- Poljak, B.T., "Subgradient Methods: A Survey of Soviet Research in Nonsmooth Optimization", Proceedings of ILASA Workshop, 1977.
- Shor, N.Z., Minimization Methods for Non-differentiable Functions, Springer-Verlag, Berlin, 1985.
- Solanki, Rajendra S. and Cohon, Jared L., "Approximating the noninferior set in linear biobjective programs using multiparametric decomposition", European Journal of Operations Research, Vol. 41, 1989.
- Steuer, Ralph E. and Schuler, Albert T., "An Interactive Multiple-Objective Linear Programming Approach to a Problem in Forest Management", Operations Research, Vol. 26, No. 2, March-April 1978.

Steuer, Ralph. E., Multiple Criteria Optimization: Theory, Computation, and Application, John Wiley & Sons, 1986.

White, D. J., "A Special Multi-Objective Assignment Problem", Journal of the Operational Research Society, Vol. 35, No.8, August 1984.

Woolsey, Robert E. D. and Swanson, Huntington S., Operations Research for Immediate Application: A Quick and Dirty Manual, Harper and Row, 1975.

# INITIAL DISTRIBUTION LIST

1. Defense Technical Information Center 2  
Cameron Station  
Alexandria, VA 22304-6145
2. Library, Code 0142 2  
Naval Postgraduate School  
Monterey, CA 93943-5002
3. Professor Siriphong Lawphongpanich, Code OR/Lp 3  
Department of Operations Research  
Naval Postgraduate School  
Monterey, CA 93943-5000
4. LCDR William Walsh, Code OR/Wa 1  
Department of Operations Research  
Naval Postgraduate School  
Monterey, CA 93943-5000
5. LT Michael S. Paul 2  
VQ-1 NAF Atsugi Japan  
FPO Seattle, WA 98767-2703
6. Navy Personnel Research and Development Center 2  
Code 112  
San Diego, CA 92152-6800
7. Commander, Naval Military Personnel Command 1  
NMPC-4  
Washinton, D.C. 20350
8. Chief of Naval Operations (OP-81) 1  
Department of the Navy  
Washington, D.C. 20350
9. Chief of Naval Operations (OP-813) 1  
Department of the Navy  
Washington, D.C. 20350
10. Commanding Officer 1  
Fleet Air Reconnaissance Squadron One (VQ-1)  
FPO San Francisco, CA 96601